EX 7 :

$$f(x)=\frac{x+2}{(x+1)^4}$$

$$f(x)=\frac{a}{x+1}+\frac{b}{(x+1)^2}+\frac{c}{(x+1)^3}+\frac{d}{(x+1)^4}$$

1. *a)* $\lim\limits_{x\to(-1)^+}(x+1)^4.f(x)=$

$$\lim_{x\to(-1)^+}(x+2)=1$$

$$\lim_{x\to+\infty}(x+1).f(x)=$$

$$\lim_{x\to+\infty}\frac{(x+1)(x+2)}{(x+1)^4}=\lim_{+\infty}\frac{x^2}{x^4}$$

$$=\lim_{+\infty}\frac{1}{x^2}=0$$

b) $(x+1)^4.f(x)=$

$$a(x+1)^3+b(x+1)^2+c(x+1)+d$$

$$\lim_{x\to(-1)^+}(x+1)^2 f(x)=d$$

or $\lim\limits_{x\to(-1)^+}(x+1)^4 f(x)=1$ *d'où* $d=1$

$$\lim_{x\to+\infty}(x+1)f(x)=$$

$$\lim_{x\to+\infty}(a+\frac{b}{x+1}+\frac{c}{(x+1)^2}+\frac{d}{(x+1)^3})=a$$

$$\lim_{x\to+\infty}(x+1)f(x)=0$$

d'où $a=0$

$$f(x)=\frac{b}{(x+1)^2}+\frac{c}{(x+1)^3}+\frac{1}{(x+1)^4}$$

2. *a)* $\dfrac{x+2}{(x+1)^4}=\dfrac{b}{(x+1)^2}+\dfrac{c}{(x+1)^3}+\dfrac{1}{(x+1)^4}$

$$\Rightarrow\frac{x+2}{(x+1)^4}-\frac{1}{(x+1)^4}=\frac{b}{(x+1)^2}+\frac{c}{(x+1)^3}$$

$$\Rightarrow\frac{1}{(x+1)^3}=\frac{b}{(x+1)^2}+\frac{c}{(x+1)^3}$$

$$\Rightarrow\frac{1-c}{(x+1)^3}=\frac{b}{(x+1)^2}$$

b) $\dfrac{1-c}{(x+1)^3}=\dfrac{bx+b}{(x+1)^3}$

$$\Rightarrow\begin{cases}b=0\\b=1-c\end{cases}\Rightarrow\begin{cases}b=0\\c=1\end{cases}$$

d'où

$$f(x)=\frac{1}{(x+1)^3}+\frac{1}{(x+1)^4}$$

3. $F(x)=\dfrac{-1}{2(x+1)^2}-\dfrac{1}{3(x+1)^3}+k$

$$F(0)=2\Rightarrow-\frac{1}{2}-\frac{1}{3}+k=0\Rightarrow k=\frac{5}{6}$$

d'où

$$F(x)=\frac{5}{6}-\frac{1}{2(x+1)^2}-\frac{1}{3(x+1)^3}$$

EX 8 :

$$f(x)=\frac{2x+1}{(x-2)^3}\quad,x\in\mathbb{R}\setminus\{2\}$$

1. $f(x)=\dfrac{2(x-2)+5}{(x-2)^3}$

$$f(x)=\frac{2}{(x-2)^2}+\frac{5}{(x-2)^3}$$

2. $F(x)=\dfrac{-2}{x-2}-\dfrac{5}{2(x-2)^2}$

$$F(x)=\frac{-4x+3}{2(x-2)^2}$$

EX 9 :

$$f(x)=\frac{1}{\sqrt{1-x^2}}\,;x\in\,]-1,1[$$

F est dérivable sur $]-1,1[$

$$\begin{cases}F'(x)=f(x)\\F(0)=0\end{cases}$$

$g(x)=F(\sin x)$